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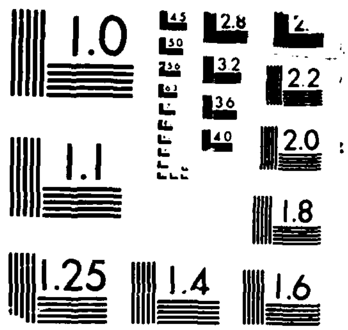

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Department of Statistics
University of North Carolina
Chapel Hill, North Carolina

LIMITING DISTRIBUTIONS OF NON-LINEAR VECTOR FUNCTIONS

OF STATIONARY GAUSSIAN PROCESSES

by

Hwai-Chung Ho

and

Tze-Chien Sun

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145. G. Kallianpur and V. Perez-Abreu, Stochastic evolution equations with values on the dual of a countably Hilbert nuclear space, July 86. Appl. Math. Optimization, to appear.

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183. H.L. Hurd, Nonparametric time series analysis for periodically correlated processes, Apr. 87.

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Hwai-Chung Ho[1]
Institute of Statistics
Academia Sinica
Taipei, Taiwan
and
Center for Stochastic Processes
Department of Statistics
University of North Carolina
Chapel Hill, NC 27599-3260

and

Tze-Chien Sun
Department of Mathematics
Wayne State University
Detroit, MI 48202

Abstract: Given a stationary Gaussian vector process (X_m, Y_m), $m \in Z$, and two real functions $H(x)$ and $K(x)$ we define $Z_H^n = A_n^{-1} \sum_{m=0}^{n-1} H(X_m)$ and $Z_K^n = B_n^{-1} \sum_{m=0}^{n-1} K(Y_m)$, where A_n and B_n are some appropriate constants. The joint limiting distribution of (Z_H^n, Z_K^n) is investigated. It is shown that Z_H^n and Z_K^n are asymptotically independent when one of them satisfies a central limit theorem. The application of this to the limiting distribution for a certain class of non-linear infinite-coordinated functions of a Gaussian process is also discussed.

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Let (X_m, Y_m), $m \in Z$, be a sequence of stationary Gaussian vectors. We assume that $EX_m = EY_m = 0$, $EX_m^2 = EY_m^2 = 1$,

$$r_1(m) = EX_oX_m \sim |m|^{-\beta_1},$$

$$r_2(m) = EY_oY_m \sim |m|^{-\beta_2},$$

as $|m| \to \infty$, and

$$r_3(m) = EX_oY_m \sim m^{-\beta_3},$$

$$r_3(-m) = EY_oX_m \sim m^{-\beta_4},$$

as $m \to \infty$, where β_1, β_2, β_3 and $\beta_4 > 0$. With their correlation functions assumed as above $\{X_m\}$ and $\{Y_m\}$ are usually called processes with long-range dependence if $\beta_1, \beta_2 < 1$. Let $G_1(x)$ and $G_2(x)$ be the spectral distributions of $\{X_m\}$ and $\{Y_m\}$, and let Z_{G_1} and Z_{G_2} be their corresponding random measures. Since $\{(X_m, Y_m)\}$ is stationary there always exists a complex-valued function $G_3(x)$ such that

$$r_3(m) = \int e^{-imx} dG_3(x), \qquad \forall m \in Z.$$

Since the matrix

$$\begin{bmatrix} G_1(dx) & G_3(dx) \\ \overline{G_3}(dx) & G_2(dx) \end{bmatrix},$$

is positive definite, it follows that

$$|G_3(dx)|^2 \le G_1(dx)G_2(dx). \tag{1}$$

Given two functions $H(x)$ and $K(x)$, satisfying $EH(X_m) = EK(Y_m) = 0$, $EH^2(X_m) < \infty$

and $EK^2(Y_m) < \infty$, and having their Hermite expansions as follows:

$$H(x) = \sum_{j=\nu_1}^{\infty} c_j H_j(x) \quad \text{and} \quad K(x) = \sum_{j=\nu_2}^{\infty} d_j H_j(x),$$

we define

$$Z_H^n = A_n^{-1} \sum_{m=0}^{n-1} H(X_m) \quad \text{and} \quad Z_K^n = B_n^{-1} \sum_{m=0}^{n-1} K(Y_m).$$

It has been proved that with proper choice of the norming factors A_n and B_n, then as $n \to \infty$, Z_H^n and Z_K^n have limiting distributions. When $\nu_i \beta_i < 1$, $i=1,2$, the limiting distribution is non-Gaussian (unless $\nu_i=1$) and can be represented by multiple Wiener integrals ([2], [8]). When the limit law is non-Gaussian, it is usually said that a non-central limit theorem (or NCLT) is satisfied. On the other hand a central limit theorem (or CLT, i.e. the norming factor is $n^{1/2}$ and the limit law is Gaussian) will hold if $\nu_i \beta_i > 1$ [1]. The purpose of this paper is to study the *joint limiting* distribution of (Z_H^n, Z_K^n), when, in particular, one component satisfies a CLT. An incomplete attempt at solving the same problem had been made by Hsiao [4].

The reason we look into this problem is the *following*: Consider the following square-integrable function $L(\cdot)$ (possibly infinite - coordinated) of a stationary Gaussian process, defined by its Wiener-Itô expansion

$$L = I_{k_1}(f_1) + I_{k_2}(f_2), \quad 1 \le k_1 < k_2,$$

where $I_j(f)$ is the j-fold multiple Wiener integral with kernel f. Let Z_L^n be defined as

$$Z_L^n = C_n^{-1} \sum_{m=0}^{n-1} I_{k_1}(U_m \circ f_1) + C_n^{-1} \sum_{m=0}^{n-1} I_{k_2}(U_m \circ f_2)$$

$$\equiv Z_{L_1}^n + Z_{L_2}^n$$

where U_m is the m-step shift operator, i.e. $(U_m \circ f)(x_1,\dots,x_k) = \exp(im(x_1 + \dots + x_k))f(x_1,\dots,x_k)$. Previous studies on the limit laws of Z_L^n are mainly directed to the cases where: both $Z_{L_1}^n$ and $Z_{L_2}^n$ satisfy either a CLT ([1], [3]) or a NCLT [7]. The case where one of $Z_{L_1}^n$ and $Z_{L_2}^n$ satisfies a CLT and the other one satisfies a NCLT is still unclear, and the following two natural questions arise: Will the limit law (if it exists) Z_L^∞ of Z_L^n be still equal to the sum of the limit laws $Z_{L_1}^\infty$ and $Z_{L_2}^\infty$ of $Z_{L_1}^n$ and $Z_{L_2}^n$; and what is the relation between $Z_{L_1}^\infty$ and $Z_{L_2}^\infty$. The main result of this paper, stated in the Theorem, provides an answer to these questions. Suppose the underlying stationary Gaussian process for Z_L^n exhibits long-range dependence. For a certain class of functions $L(\cdot)$, by making use of the formula for the change variables ([5], p. 32) on the kernels f_1 and f_2, we may obtain

$$(Z_{L_1}^n, Z_{L_2}^n) \stackrel{\Delta}{=} (C_n^{-1} \sum_{m=0}^{n-1} H_{k_1}(Y_m'), \quad C_n^{-1} \sum_{m=0}^{n-1} H_{k_2}(X_m')),$$

for some sequence of stationary Gaussian vectors (X_m', Y_m'), $m \in Z$. " $\stackrel{\Delta}{=}$ " means equal in distribution. Assume $C_n = n^{1/2}$. Suppose $Z_{L_1}^n$ and $Z_{L_2}^n$ satisfy a CLT and a NCLT respectively. If, furthermore, the conditions in the Theorem are met by $\{(X_m', Y_m')\}$, then as a result of the Theorem, it follows that

$$Z_{L_1}^\infty \perp Z_{L_2}^\infty \quad \text{and} \quad Z_L^\infty \stackrel{\Delta}{=} Z_{L_1}^\infty + Z_{L_2}^\infty$$

("$\perp$" means independent), i.e. the distribution function $\tilde{L}(x)$ of Z_L^∞ can be written as

$$\tilde{L}(x) = \int F(x-y)d\Phi(y/\sigma), \qquad \sigma > 0,$$

for some distribution function F(y) and a standard Gaussian distribution $\Phi(y)$. It should be pointed out that a NCLT with norming factor $n^{1/2}$, such as for $Z^n_{L_2}$, is shown possible in [6]. A more detailed study of the situation where a CLT and a NCLT jointly occur will appear in a subsequent paper by the authors. Now we formulate our main result:

<u>Theorem</u>. Assume $\nu_1\beta_1 < 1 < \nu_2\beta_2$. When $\nu_2 = 1$ we also assume

$$\beta \equiv \beta_3 \wedge \beta_4 > \frac{1 + \beta_1}{2} .$$

Then with $A_n = n^{1-\nu_1\beta_1/2}$ and $B_n = n^{1/2}$ the limiting distributions Z^*_H and Z^*_K of Z^n_H and Z^n_K are independent.

Note that Z^*_k is Gaussian by [1].

Throughout the rest of the paper we always assume $\nu_1\beta_1 < 1$ and $\nu_2\beta_2 > 1$. Later, in proving the Theorem, we shall only deal with the very special case where H(x) and K(x) have the following one-term expansion

$$H(x) = H_{\nu_1}(x) \quad \text{and} \quad K(x) = H_{\nu_2}(x).$$

The reduction of H(x) to its first term is justified because when $\nu_1\beta_1 < 1$ only the first term is relevant to the distribution Z^*_H [8]. In [1] it is made clear that when $\nu_2\beta_2 > 1$ we need only to consider the K(x) with finite expansion to prove the central limit theorem. Though we prove the Theorem only for the K(x) with one-term expansion, the arguments in the proof can be easily extended to the finite expansion case.

The major tool we use to prove the Theorem is the so-called "diagram formula" [5] on how to compute the expectation of a product of Hermite polynomials of standard Gaussian random variables. Prior to giving the statement of the formula, we need some notations and definitions. Let a given set of $(\ell_1 + \dots + \ell_p)$ vertices be arranged into p levels such that the i-th level has ℓ_i vertices. A graph G is called a diagram of order $(\ell_1, \dots, \ell_p)$ if (1) each vertex is of degree one and (2) edges may pass only between different levels. By a regular diagram we mean a diagram whose edges do not pass between levels in different pairs. For each edge $w \in G$ connecting the i-th and j-th level, $i < j$, define $d_1(w) = i$ and $d_2(w) = j$.

<u>Lemma 1</u>. (Diagram Formula) Let $(W_1, \dots, W_p)$ be a Gaussian vector with $EW_i = 0$, $EW_i^2 = 1$, and $EW_iW_j = r(i,j)$. Then for the Hermite polynomials $H_{\ell_1}(x), \dots, H_{\ell_p}(x)$, we have

$$E \prod_{i=1}^{p} H_{\ell_i}(W_i) = \sum_G \prod_{w \in G} r(d_1(w), d_2(w)),$$

where the sum runs through all the diagrams G of order $(\ell_1, \dots, \ell_p)$.

The following lemma is well-known and can be easily derived from Lemma 1.

<u>Lemma 2</u>. Given two r.v.'s Z and W with $EZ = EW = 0$, $EZ^2 = \sigma_1^2$ and $EW^2 = \sigma_2^2$, then Z and W are independent Gaussian r.v.'s if and only if for all ℓ, m,

$$EZ^\ell W^m = \begin{cases} \dfrac{\ell!\ m!}{2^{\ell/2+m/2}(\ell/2)!(m/2)!} \sigma_1^\ell \sigma_2^m & \text{if } \ell \text{ and } m \text{ are even} \\ 0 & \text{otherwise.} \end{cases}$$

In the following "Δ" always denotes bounded Borel sets in $\mathbb{R}$. Then because $r_3(n) = \int e^{-inx} dG_3(x)$, we have

$$E\int f(x) Z_{G_1}(dx) \int g(x) Z_{G_2}(dx) = \int f(x)\bar{g}(x) dG_3(x) \tag{2}$$

for $f \in L^2(G_1)$ and $g \in L^2(G_2)$. By Proposition 1 of [2] or similar arguments it can be proved that there exist $G_1^*(x)$ and $G_2^*(x)$ such that

$$n^{\beta_1} G_1(\tfrac{dx}{n}) \xrightarrow{\text{weakly}} G_1^*(dx) \tag{3.1}$$

and

$$n^{\beta_2^*} (\log n)^{-\delta(\beta_2)} G_2(\tfrac{dx}{n}) \xrightarrow{\text{weakly}} G_2^*(dx) \tag{3.2}$$

as $n \to \infty$, where $\beta_2^* = \beta_2 \wedge 1$ and $\delta(x) = 1$ if $x=1$, and $= 0$ if $x \neq 1$.

We shall need the following lemma to prove the Theorem. Recall that $\beta=\beta_3 \wedge \beta_4$.

Lemma 3. Assume $\beta \leq 1$. There exists a function $G_3^*(x)$ of locally bounded variation such that for each bounded Borel set Δ,

$$\lim_{m\to\infty} m^{\beta} (\log m)^{-\delta(\beta)} G_3(\tfrac{\Delta}{m}) = G_3^*(\Delta). \tag{4}$$

Moroever G_3^* satisfies

$$G_3^*([0,y]) = \overline{G_3([-y,0])} = y^{\beta} D,$$

where D is some complex constant.

<u>Proof</u>: It is sufficient to show that (4) holds for $\Delta = [0,y]$ or $[-y,0]$. Define

$$F_n(x) = \frac{1}{2\pi} \sum_{|s|\leq n} r_3(x) \cdot \int_{-\pi}^{x} e^{isy} dy$$

for $x \in [-\pi,\pi]$. Since each term in the above sum is bounded by $C\cdot|s|^{-\beta-1}$ for some constant C. $F_n(x)$ converges to $G_3(x)$ for all x. i.e.

$$G_3(x) = \lim_{n\to\infty} F_n(x) = \frac{1}{2\pi} \sum_{s=-\infty}^{\infty} r_3(s) \quad \frac{e^{isx} - e^{-is\pi}}{is}.$$

Let $\Delta = [0,y]$. Define

$$S_{m,y} = m^{\beta}(\log m)^{-\delta(\beta)}[G_3(\tfrac{\Delta}{m}) + \overline{G_3(\tfrac{\Delta}{m})}].$$

By $\overline{G_3(\Delta)} = G_3(-\Delta)$, $S_{m,y}$ is equal to

$$S_{m,y} = \frac{1}{\pi}(\log m)^{-\delta(\beta)} \sum_{s=-\infty}^{\infty} (r_3(s)m^{\beta}) \frac{\sin \frac{s}{m} y}{s},$$

which as $m \to \infty$ tends to

$$\lim_{m\to\infty} S_{m,y} = \begin{cases} y^{\beta}R_{\beta} & \text{if } \beta_3 \neq \beta_4, \\ y^{\beta}2R_{\beta} & \text{if } \beta_3 = \beta_4, \end{cases} \tag{4.1}$$

where $R_{\beta} = \frac{1}{\pi}\lim_{\epsilon\to 0}(-\log \epsilon)^{-\delta(\beta)} \int_{\epsilon}^{\infty} x^{-\beta-1}\sin x\, dx$. Similarly if we define

$$C_{m,y} = m^{\beta}(\log m)^{-\delta(\beta)}[G_3(\tfrac{\Delta}{m}) - \overline{G_3(\tfrac{\Delta}{m})}]$$

$$= i\,\frac{1}{\pi}(\log m)^{-\delta(\beta)} \sum_{s=-\infty}^{\infty} (r_3(s)m^{\beta}) \frac{(1-\cos \frac{s}{m} y)}{s}$$

then we obtain

$$\lim_{m\to\infty} C_{m,y} = \begin{cases} -iy^{\beta}I_{\beta} & \text{if } \beta_3 < \beta_4 \\ iy^{\beta}I_{\beta} & \text{if } \beta_4 < \beta_3 \\ 0 & \text{if } \beta_3 = \beta_4 \end{cases}, \tag{4.2}$$

where $I_{\beta} = \frac{1}{\pi}\lim_{\epsilon\to 0}(-\log \epsilon)^{-\delta(\beta)} \int_{\epsilon}^{\infty} x^{-\beta-1}(1-\cos x)dx$. (4.1) and (4.2) imply

$$\lim_{m\to\infty} m^{\beta}(\log m)^{-\delta(\beta)}G_3(\frac{[0,y]}{m}) = y^{\beta}D \equiv G_3^{*}([0,y]),$$

where

$$D \equiv \begin{cases} y^{\beta}R_{\beta} & \text{if } \beta_3 = \beta_4 \\ y^{\beta}(R_{\beta} - iI_{\beta})/2 & \text{if } \beta_3 < \beta_4 \\ y^{\beta}(R_{\beta} + iI_{\beta})/2 & \text{if } \beta_4 < \beta_3 \end{cases} .$$

Since the property $\overline{G_3(\Delta)} = G_3(-\Delta)$ is preserved by passing to the limit G_3^*, we have

$$G_3^*([-y,0]) = y^{\beta}\overline{D}.$$

The proof is completed. □

Assume $\beta \leq 1$ again and observe that

$$m^{\beta}(\log m)^{-\delta(\beta)} \ |G_3(\tfrac{\Delta}{m})|$$

$$\leq m^{\beta-(\beta_1+\beta_2^*)/2}(\log m)^{\delta(\beta_2)-\delta(\beta)}[m^{\beta_1}G_1(\tfrac{\Delta}{m})m^{\beta_2^*}(\log m)^{-\delta(\beta_2)}G_2(\tfrac{\Delta}{m})]^{1/2}.$$

Then we have an immediate corollary from (3.1), (3.2) and (4):

$$\beta \geq (\beta_1 + \beta_2^*)/2. \tag{4.3}$$

When $\beta > 1$, then $G_3(dx)$ is absolutely continuous and its density is continuous. Let $G_3(dx)=f(x)dx$. Then

$$\lim_{m\to\infty} mG_3(\tfrac{\Delta}{m}) = \lambda(\Delta)\ f(0) , \tag{4.4}$$

where λ is the Lebesgue measure. (4.3) is clearly satisfied for $\beta > 1$.

By (3.1),

$$n^{1-\beta_1/2}Z_{G_1}(\tfrac{\Delta}{n}) \xrightarrow{d} Z_{G_1^*}(\Delta)$$

as $n \to \infty$ [2], where $Z_{G_1^*}$ is the random measure induced by $G_1^*(dx)$. Since the distribution of Z_H^* can be represented by the υ_1-fold Wiener integral, we need to show that for disjoint Δ_i's, $i=1,2,\ldots,\upsilon_1$,

$$(Z_{G_1}^*(\Delta_1),\ldots,Z_{G_1}^*(\Delta_{\upsilon_1})) \perp Z_k^* ,$$

which is in fact equivalent to showing for each Δ,

$$Z_{G_1}^*(\Delta) \perp Z_k^* . \tag{5}$$

It is not difficult to see that (5) is equivalent to

$$W(\Delta) = \int_\Delta \frac{e^{ix} - 1}{ix} Z_{G_1}^*(dx) \perp Z_k^* . \tag{5.1}$$

It is mere technical convenience that leads us to replace $Z_{G_1^*}(\Delta)$ by $W(\Delta)$.

Define

$$K_n(x) \equiv \frac{e^{ix} - 1}{(e^{ix/n} - 1)n} = \frac{1}{n} \sum_{j=0}^{n-1} e^{ijx/n} \tag{5.2}$$

and

$$W_n(\Delta) \equiv n^{-(1-\beta_1/2)} \sum_{j=0}^{n-1} \int_{\frac{\Delta}{n}} e^{ijx} Z_{G_1}(dx)$$

$$= \int_\Delta K_n(x)\, n^{-(1-\beta_1/2)} Z_{G_1}(\tfrac{dx}{n}) .$$

(3.1) and the fact that $K_n(x)$ converges to $(e^{ix}-1)/ix$ uniformly on every bounded set imply

$$W_n(\Delta) \xrightarrow{d} W(\Delta) \tag{6}$$

as $n \to \infty$ [2]. If we can show

$$\lim_{n\to\infty} E(W_n(\Delta))^{\ell}(Z_k^n)^m$$

$$(7)\qquad = \begin{cases} \dfrac{\ell!\ m!}{2^{\ell/2 + m/2}(\ell/2)!\ (m/2)!}\sigma_1^{\ell}\sigma_2^{m} & \text{if } \ell \text{ and } m \text{ are even,} \\ 0 & \text{otherwise} \end{cases} .$$

then by (6) and Lemma 2, (5.1) follows. Notice that $EW^2(\Delta) \equiv \sigma_1^2$ and $E(Z_k^*)^2 = \sigma_2^2$.

Proof of Theorem.

Given a fixed setting of vertices $V = (1,\ldots,1,\ \nu_2,\ldots,\nu_2)$ having as follows its configuration

$$\begin{array}{cc} \ell & \left\{ \begin{array}{l} 0 \\ 0 \\ \vdots \\ 0 \end{array} \right. \\ m & \left\{ \begin{array}{cccc} 0 & 0 & \ldots & 0 \\ 0 & 0 & \ldots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \ldots & 0 \end{array} \right. \\ & \nu_2 \end{array}$$

Define Γ = the set of all regular diagrams of order V and Γ^c the complement of Γ, i.e., the set of all non-regular diagrams of order V. Any subgraph of a diagram is called a subdiagram if it is itself a diagram and is the union of levels and the edges on the levels. Any diagram $G \in \Gamma^c$ can be partitioned into three disjoint subdiagrams $V_{G,1}$, $V_{G,2}$ and $V_{G,3}$, which are defined as follows.

$V_{G,1}$ = the maximal subdiagram of G which is regular within itself, and all its edges satisfy $1 \le d_1(w) < d_2(w) \le \ell$ or $\ell + 1 \le d_1(w) < d_2(w) \le \ell + m$.

$V_{G,2}$ = the maximal subdiagram of $G - G_{G,1}$ whose edges satisfy $\ell + 1 \le d_1(w) < d_2(w) \le \ell + m$.

$V_{G,3} = G - (V_{G,1} \cup V_{G,2})$.

For each subdiagram $V_{G,i}$ of G, i=1,2,3, define

$$V^*_{G,i} = \{j \mid \text{the } j\text{-th level of } V \text{ is in } V_{G,i}\}.$$

$$V^*_{G,i}(1) = \{j \in V^*_{G,i} \mid 1 \le j \le \ell\}.$$

$$V^*_{G,I}(2) = \{j \in V^*_{G,i} \mid \ell + 1 \le j \le \ell + m\}.$$

In the following E(G) denotes the set of all edges contained in the diagram G.

Use Lemma 2 (Diagram Formula)

$$E(W_n(\Delta))^{\ell}(Z^n_k)^m = \sum_{G \in \Gamma} [E(W_n(\Delta))^2]^{\ell/2}[E(Z^n_k)^2]^{m/2}$$

$$+ \sum_{G \in \Gamma^c} [\prod_{w \in E(V_{G,i})} E(W_n(\Delta))^2 \prod_{w \in E(V_{G,1})} E(Z^n_k)^2]$$

$$\times [n^{-|V^*_{G,2}(2)|/2} \sum_{\substack{0 \le p_i \le n-1 \\ i \in V^*_{G,2}}} \prod_{w \in E(V_{G,2})} r_2(p_{d_1(w)} - p_{d_2(w)})]$$

$$\times [n^{-|V^*_{G,3}(2)|/2} \sum_{\substack{0 \le p_i \le n-1 \\ i \in V^*_{G,3}}} \prod_{\substack{w \in E(V_{G,3}) \\ d_1(w) \in V^*_{G,3}(2)}} r_2(p_{d_1(w)} - p_{d_2(w)}) \cdot$$

$$\cdot \prod_{\substack{w \in E(V_{G,3}) \\ d_1(w) \in V^*_{G,3}(1)}} n^{-(1-\beta_1/2)} E(\int_{\frac{\Delta}{n}} \exp(ip_{d_1(w)}x) Z_{G_1}(dx) \int \exp(ip_{d_2(w)}x) Z_{G_2}(dx)]$$

(8) $$\equiv \Sigma^n_\Gamma + \sum_{G\in\Gamma^c} A_1^n \times A_2^n \times A_3^n .$$

Since Σ^n_Γ converges to the right hand side of (7), it is sufficient to show that for fixed $G \in \Gamma^c$ the second term of (8) vanishes.

(9) $$\lim_{n\to\infty} A_1^n \times A_2^n \times A_3^n = 0.$$

Recall the definition of $W_n(\Delta)$. We have

(10) $$EW_n^2(\Delta) \to \int_0 \left| \frac{e^{ix} - 1}{ix} \right|^2 dG_1^*(x) = EW^2(\Delta).$$

as $n \to \infty$. (10) and the central limit theorem for Z_k^n imply

(11) $$\lim_{n\to\infty} A_1^n = (EW^2(\Delta))^{|V^*_{G,1}(1)|/2} (\sigma_2^2)^{|V^*_{G,1}(2)|/2} .$$

Using (2) and (5.2), we can rewrite

(12) $$A_3^n = n^{-|V^*_{G,3}(2)|/2} \sum_{\substack{0\le p_i\le n-1 \\ i\in V^*_{G,3}(2)}} \prod_{\substack{w\in E(V_{G,3}) \\ d_1(w)\in V^*_{G,3}(2)}} r_2(p_{d_1(w)} - p_{d_2(w)}) \cdot$$

$$\cdot \prod_{\substack{e\in E(V_{G,3}) \\ d_1(e)\in V^*_{G,3}(1)}} \Big[\sum_{0\le p_{d_1(e)}\le n-1} n^{-(1-\beta_1/2)} \int_\Delta e^{i\left(\frac{p_{d_1(e)} - p_{d_2(e)}}{n}\right)x} dG_3\left(\tfrac{x}{n}\right)\Big].$$

Fix p_i, $i\in V^*_{G,3}(2)$, and $e \in E(V_{G,3})$ with $d_1(e) \le \ell$, we obtain as a result of (4) in Lemma 3 (or (4.4) if $\beta > 1$) an asymptotic bound for the second summation (denoted by Σ^*_n) in (12). In the following $\alpha \equiv \beta \wedge 1$.

$$\Sigma_n^* = n^{(\beta_1-2\alpha)/2}(\log n)^{\delta(\beta)} \cdot \int_\Delta [\sum_{0\le p_{d_1(e)}\le n-1} \exp(ip_{d_1(e)}x/n) \cdot \frac{1}{n}] \cdot$$

$$\cdot \exp(-ip_{d_2(e)}x/n) \cdot n^\alpha(\log n)^{-\delta(\beta)}dG_3(\frac{x}{n}).$$

$$= O(n^{(\beta_1-2\alpha)/2}(\log n)^{\delta(\beta)}\int_\Delta \left| \frac{e^{ix}-1}{ix} \right| |dG_3^*(x)|).$$

If $\beta \le 1$, then

(13) $$\beta_1 - 2\alpha \le \beta_1 - 2\beta \le -\beta_2^* < 0,$$

where we make use of the fact that $\beta \ge (\beta_1 + \beta_2^*)/2$ derived right after Lemma 3. If $\beta > 1$, clearly (13) still holds. By (13) it follows that

(13.1) $$\Sigma_n^* = o(1).$$

Define

$k(i)$ = the number of edges w satisfying $d_1(w) = i$.

and

$g(i)$ = the number of vertices in the i-th level not connecting any of the first ℓ levels.

We firstly assume that $\nu_2 > 1$. By the similar argument employed to prove Proposition in [1], we can develop the following facts:

(14) $$\lim_{n\to\infty} A_2^n = 0$$

if $V_{G,2}$ is nonempty. And secondly an asymptotic bound as (17) can be obtained for the first summation in (12) if $V_{G,3} \ne \phi$.

$$\alpha_n \equiv \sum_{\substack{0\le p_i\le n-1\\ i\in V^*_{G,3}(2)}} \prod_{\substack{w\in E(V_{G,2})\\ d_1(w)\in V^*_{G,3}(2)}} |r_2(p_{d_1(w)} - p_{d_2(w)})| \tag{15}$$

$$= O\left(n^{|V^*_{G,3}(2)| - \sum_{i\in V^*_{G,3}(2)} \frac{k(i)}{g(i)}}\right)$$

Note that the α_n given above is well-defined because it is assumed that $\nu_2>1$. As shown in (2.20) in [1] we have the following inequality

$$\sum_{i\in V^*_{G,3}(2)} \frac{k(i)}{g(i)} \geq \frac{1}{2}\, |V^*_{G,3}(2)| \;. \tag{16}$$

(15) and (16) imply that

$$\alpha_n = O(n^{|V^*_{G,3}(2)|/2}) \;. \tag{17}$$

Then (12), (13.1) and (17) imply

$$A_3^n = (o(1))^{|V^*_{G,3}(1)|} \;.$$

Hence if $|V^*_{G,3}(1)| > 0$ (because $V_{G,3} \neq \phi$), then

$$\lim_{n\to\infty} A_3^n = 0 \;. \tag{18}$$

For any non-regular diagram $G \in \Gamma^c$ if $\nu_2 > 1$ then its subdiagrams $V_{G,2}$ and $V_{G,3}$ can not be empty at the same time, that is either (14) or (18) must hold. Hence (9) is true.

When $\nu_2 = 1$, then $V_{G,2}$ is empty, i.e. A_2^n is absent. Thus in order to assure (9) we have to show (18). Also note that when $\nu_2 = 1$ the first product in (12) no longer exists. Rewrite A_3^n given in (12) in a more simplified form and apply to it the result of Lemma 3. We have

$$A_3^n = \prod_{w \in E(V_{G,3})} n^{(\beta_1+1)/2-\alpha} (\log n)^{\delta(\beta)}$$

$$\cdot \int_\Delta [\sum_{0 \le p_{d_1(w)} \le n-1} \exp(ip_{d_1 w}\, x/n) \cdot \frac{1}{n}] \cdot$$

$$\cdot [\sum_{0 \le p_{d_2(w)} \le n-1} \exp(-ip_{d_2(w)} x/n) \cdot \frac{1}{n}] \, n^{\alpha} (\log n)^{-\delta(\beta)} dG_3(\frac{x}{n})$$

$$= O(n^{(\beta_1+1)/2-\alpha} (\log n)^{\delta(\beta)} \int_\Delta | \frac{e^{ix} - 1}{ix} |^2 \, |dG_3^*(x)|).$$

By the assumption of the Theorem, when $\upsilon_2 = 1$,

$$\frac{\beta_1 + 1}{2} < \beta \Rightarrow \frac{\beta_1 + 1}{2} - \alpha < 0.$$

$$\Rightarrow \lim_{n \to \infty} A_3^n = 0.$$

The proof is completed. $\square$

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